Homework 1

Due: Thursday April 11, 2013

1. Find the greatest common divisor of 1112 and 1544.

2. For the value d of the greatest common divisor found in the first question, find all integer solutions (x, y) to the equation $1112x + 1544y = d$

3. Find all solutions of the congruences $12x \equiv 28 \pmod{236}$ and $12y \equiv 30 \pmod{236}$.

4. Find a multiplicative inverse of 7 modulo 30. (ie x such that $7x \equiv 1 \pmod{30}$).

5. Let p be a prime number and n a positive integer. Show that the largest power of p which divides $n!$ is given by
$$\sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor .$$
(Here $\lfloor x \rfloor$ is the largest integer not greater than x).

6. Prove that the binomial coefficient $\binom{2n}{n} = \frac{(2n)!}{n!n!}$ divides the product
$$\prod_p p^{\lfloor \log_p(2n) \rfloor}$$
where the product is taken over all primes p. (For a hint, what possible values can $\lfloor 2x \rfloor - 2\lfloor x \rfloor$ take?)