# Homework 4

## Due: Thursday May 2, 2013

1. Let $f(z)$ be a meromorphic function. Find all residues of the function $F(z) = f'(z)/f(z)$ (in terms of the zeros and poles of $f$).

2. The ordered Bell numbers $b_n$ (which count the number of weak orderings (ties allowed) of a set of $n$ elements) have exponential generating function

$$\sum_{n=0}^{\infty} \frac{b_n}{n!} z^n = \frac{1}{2 - e^z}.$$

   Use this to derive an asymptotic formula for $b_n$.

3. Let $t_n$ denote the number of involutions in the symmetric group $S_n$. This has exponential generating function

$$\sum_{n=0}^{\infty} \frac{t_n}{n!} z^n = e^{z + z^2/2}.$$

   Prove the asymptotic formula

$$t_n \sim \sqrt{\frac{n^n}{2}} \exp(-n/2 + \sqrt{n} - 1/4)$$

4. Let $\{z_n\}_{n=1}^{\infty}$ be a sequence of complex numbers with $\sum_{i=1}^{\infty} |z_i|$ convergent. Prove that the infinite product $\prod_{i=1}^{\infty}(1 + z_i)$ converges and is nonzero if and only if $z_i \neq -1$ for all $i$.